

Anthony W. Thomas · Phiala E. Shanahan · Ross  
D. Young

# Strange quarks and lattice QCD

Received: date / Accepted: date

**Abstract** The last few years have seen a dramatic improvement in our knowledge of the strange form factors of the nucleon. With regard to the vector form factors the level of agreement between theory and experiment gives us considerable confidence in our ability to calculate with non-perturbative QCD. The calculation of the strange scalar form factor has moved significantly in the last two years, with the application of new techniques which yield values considerably smaller than believed for the past 20 years. These new values turn out to have important consequences for the detection of neutralinos, a favourite dark matter candidate. Finally, very recent lattice studies have resurrected interest in the famed H-dibaryon, with modern chiral extrapolation of lattice data suggesting that it may be only slightly unbound. We review some of the major sources of uncertainty in that chiral extrapolation.

## 1 Introduction

One of the major initial successes of QED was the successful explanation of the famous Lamb shift in terms of the effect of vacuum polarization. For QCD the strange form factors of the nucleon occupy a position of comparable importance. While lattice QCD has accurately described a number of valence quark dominated hadronic properties, the strange form factors of the nucleon can only arise through quantum fluctuations in which a strange-anti-strange pair briefly bubble into and out of existence.

It is now more than 20 years since it was realized that parity violating electron scattering (PVES) could provide a third, independent constraint on the vector form factors of the nucleon, thus allowing one to solve for the strange vector matrix elements [1]. Following a series of state-of-the-art measurements at MIT-Bates, Mainz and JLab (for a recent review see Ref. [2]), as well as systematic study of the relevant radiative corrections [3] and a careful global analysis [4], we now know that the strange magnetic form factor is at most a few percent of the proton magnetic form factor at low- $Q^2$  and the strange electric radius is also at most a few percent of the proton charge radius.

The length of time since the initial studies of nucleon vector form factors in lattice QCD, by Leinweber and collaborators, is also around 20 years. In combination with chiral extrapolation of lattice data using finite range regularisation, indirect techniques developed by Leinweber and Thomas [5] led to a very precise determination of both the strange magnetic moment of the proton and its strange charge radius [6] some 15 years later. Although it initially seemed as though these calculations disagreed with the PVES data, it is now clear that the agreement is excellent. Furthermore, precise, direct calculations of these form factors from the Kentucky group in just the last two years [7] agree very well with the results of the earlier indirect work and with the experimental results.

At present, the accuracy of the theoretical calculations exceeds that of the best experiments by almost an order of magnitude – a remarkable exception in strong interaction physics. Clearly the challenge is there for a clever new idea to take us beyond the current experimental limitations. Nevertheless,

this future challenge should not blind us to the tremendous achievements thus far and especially to the fact that QCD has passed its equivalent of the “Lamb shift test” with flying colours.

In the next section we briefly review the state of play with respect to the strange sigma commutator (the strange scalar form factor), for which the best value has taken a major shift in the last two or three years. We also point out the relevance to dark matter searches. In the third section we recall the recent surprises concerning the H-dibaryon and after recalling the best current estimate of its mass we explore some of the potential uncertainties.

## 2 Strange scalar form factor

In many ways it is fortunate that we have not had sufficient computing power to directly calculate hadron properties at the physical quark masses. Instead, we have been given data on a number of hadronic properties *with QCD* as a function of the masses of the quarks. This is of course information that Nature cannot give us but which is nevertheless an invaluable guide to how QCD actually works [8]. From those studies it is clear that for whatever reason (and we suggest one below) the properties of baryons and non-Goldstone mesons made of light quarks behave exactly as one would expect in a constituent quark model once the pion mass is above about 0.4 GeV. That is, all the famous, rapid, non-analytic variation associated with Goldstone boson loops is seen to disappear in this region. There has been some speculation that this scale (which corresponds to a quark mass around 40 MeV) may have something to do with the size of an instanton. However, it is clear that such behaviour does emerge naturally if one takes into account the finite size of such hadrons, which necessarily suppresses meson loops at large momentum transfer and at large mass for the Goldstone bosons. Put simply, meson loops are suppressed when the corresponding Compton wavelength is smaller than the size of the hadron emitting or absorbing the meson. Given that a typical hadron size is 1fm, and the Compton wavelength of a meson of mass 0.4 GeV only 0.5fm, one has a natural explanation.

This qualitative lesson from QCD itself, leads us to anticipate that the contribution of strange quarks to nucleon properties should be suppressed, as the mass of the kaon is 0.5 GeV – above the critical scale.

This idea has also been developed to allow quantitative advances in the calculation of hadron properties using finite range regularization (FRR) [9]. This technique allows one to effectively resum the chiral expansion of hadronic properties and to accurately describe the variation of properties such as the mass of the baryon octet over a much larger range of quark masses than expected within naive chiral perturbation theory.

Once one has an accurate parametrization of the mass of the nucleon as a function of pion and kaon mass, based on a fit to modern lattice data for the nucleon octet using FRR, it is trivial to extract the light quark sigma commutators by differentiation, using the Feynman-Hellmann theorem. It is this approach which has recently established that  $\sigma_s$  is almost an order of magnitude smaller than had been generally believed for 20 years [10]. Very similar conclusions have been reached by several other modern lattice simulations [11, 12, 13], with everyone agreeing that the value is between 20 and 50 MeV. Although at first sight it is shocking that there can be such a large shift in a fundamental property of the nucleon, it is quite common with fundamental parameters that apparent convergence can be followed by a shift by far more than the quoted uncertainties when a new technique becomes available.

The importance of this new value for kaon condensation in dense matter remains to be investigated. It is in the search for dark matter that the new value of  $\sigma_s$  has had its immediate impact [14]. In the minimal supersymmetric extension of the Standard Model, constrained by all particle physics and WMAP data, the so-called CMSSM, the favoured candidate for dark matter is the neutralino [16], a weakly interacting fermion with mass of order of a hundred GeV or more. With the old values of  $\sigma_s$  its dominant interaction with the nucleon was through the strange quark – which had led to Ellis and collaborators calling desperately for a more accurate determination of the strange quark sigma commutator [15]. The new value reported above not only reduces the expected cross section by an order of magnitude compared with earlier optimistic expectations but it also makes them far more accurate. We refer to the work of Giedt *et al.* for more details [14].

**Table 1** Values of fit parameters for the octet and  $H$ -dibaryon data with monopole, exponential and sharp cutoff regulators used. Note that units are given in dimensionally appropriate powers of GeV.

Monopole	$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$	$\sigma$	$B_0$	$\sigma_B$	$C_H$
best fit value	0.63	0.87	-1.66	-1.16	-0.49	0.019	-2.27	5.56
error	0.04	0.04	0.11	0.09	0.05	0.004	0.19	0.07
<b>Exponential</b>								
best fit value	0.81	0.86	-1.75	-1.22	-0.53	0.020	-2.44	5.71
error	0.05	0.04	0.11	0.10	0.05	0.004	0.21	0.10
<b>Sharp cutoff</b>								
best fit value	0.57	0.83	-1.94	-1.36	-0.61	0.018	-2.81	5.95
error	0.02	0.03	0.10	0.08	0.05	0.003	0.18	0.10

### 3 Uncertainties in the estimate of the mass of the $H$ -dibaryon

In this section we turn to the extremely exciting possibility that the  $H$ -dibaryon, which had been abandoned by many, may indeed be almost bound with respect to the  $\Lambda - \Lambda$  threshold. This possibility was raised only in the last year by the NPLQCD and HAL collaborations [17, 18], which found the  $H$  bound by tens of MeV at the unphysically large light quark masses where they could perform the calculations. The issue then becomes whether one can accurately extrapolate these results to the physical quark masses. Given our explanation of the effectiveness of FRR and the relatively large quark masses at which much of the data was taken, this seems the natural technique to apply. It is helpful that the corresponding chiral coefficient for the  $H$ , assuming that it is indeed a genuine multi-quark state and not a quasi-molecular state like the deuteron, were calculated in the early 80s by Mulders and Thomas [19].

By making a simultaneous fit to the precise octet baryon masses noted earlier, as well to the data for the  $H$ , Shanahan *et al.* were able to show that at the physical quark masses the  $H$  is most likely slightly unbound:  $m_H - 2m_\Lambda = 13 \pm 14$  MeV [20]. In what follows we explore, in more detail than was possible in the original article, the uncertainties in this result. These turn out to be rather small.

#### 3.1 Variation of the form of the regulator

We investigate the claim that varying the form of the regulator,  $u(k)$ , used with the FRR formalism does not change the results obtained for the binding of the  $H$ -dibaryon. As well as the dipole form used for the primary fit, we investigate monopole and exponential forms:

$$\text{monopole :} \quad u_m(k) = \frac{\Lambda^2}{\Lambda^2 + k^2} \quad (1)$$

$$\text{exponential :} \quad u_e(k) = \exp\left(\frac{-k^2}{\Lambda^2}\right). \quad (2)$$

A sharp cut-off was also tested. Fits with each of these regulators yield  $\chi^2$  values (per degree of freedom) between 0.48 and 0.49, with the  $H$  unbound at the physical point by  $13 \pm 14$  MeV (monopole and exponential forms), or  $14 \pm 16$  MeV (sharp cutoff). Octet masses from each of these fits match the results obtained from the fit with a dipole regulator to within the quoted precision. Best-fit parameter values for these fits are given in Table 1. It is clear that the form of the regulator does not significantly effect the results, and the selection of a dipole form as regulator is justifiable.

#### 3.2 Variation of form factor mass for the $H$ dibaryon

In the primary fit, the regulator mass  $\Lambda$  was taken to be the same for both the  $\Lambda$  hyperon and the  $H$ -dibaryon. When the ratio  $\frac{\Lambda_H}{\Lambda_\Lambda}$  is varied, however, we do find a significant variation in binding, with little associated variation in the quality of the fit. For  $\Lambda_H$  20% smaller than  $\Lambda_\Lambda$ , the  $H$  is bound at the

**Table 2** Values of fit parameters for the octet and  $H$ -dibaryon data, with  $\Lambda_H$  20% smaller or larger than  $\Lambda_\Lambda$ . All quantities are given in appropriate powers of GeV.

		$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$	$\sigma$	$B_0$	$\sigma_B$	$C_H$
$\frac{\Lambda_H}{\Lambda_\Lambda}=0.8$	best fit value	1.02	0.86	-1.71	-1.20	-0.51	0.08	-3.41	10.03
	error	0.06	0.04	0.12	0.10	0.05	0.01	0.26	0.50
$\frac{\Lambda_H}{\Lambda_\Lambda}=1.2$	best fit value	1.01	0.86	-1.70	-1.19	-0.51	-0.02	-1.76	3.69
	error	0.06	0.04	0.11	0.09	0.05	0.01	0.15	0.05

physical point by  $2 \pm 14$  MeV. Note, however, that in this case the best-fit value of the chiral coefficient of the  $H$ , called  $C_H$ , grows to approximately 2.5 times the Mulders-Thomas estimate [19]. For  $\Lambda_H$  20% larger than  $\Lambda_\Lambda$ , the  $H$  is unbound by  $23 \pm 13$  MeV. In this case, the best-fit value of  $C_H$  in fact moves closer to the Mulders-Thomas estimate; to within 10%. The  $\chi^2$  per degree of freedom for these fits are 0.47 and 0.49, respectively, a comparable quality of fit to the primary (which had  $\chi^2/\text{dof} = 0.48$ ). Best-fit values are shown in Table 2.

While naively one may expect that  $\Lambda_H$  should be somewhat smaller than  $\Lambda_\Lambda$ , corresponding to the expectation that the  $H$  should be larger than the  $\Lambda$  baryon in coordinate space, this is obscured by the regularisation scheme. As a different choice of  $\Lambda$  corresponds to a difference in the resummation of higher-order terms in the formal chiral expansion, it is likely that the dominant sensitivity of the fit to a variation in  $\Lambda$  will vanish, being compensated for by variations of the fit parameters  $B_0$  and  $\sigma_B$ .

For this reason, we suggest that, even though the inclusion of a mass parameter  $\Lambda$  was motivated by the physical idea that loop processes with momenta greater than  $\Lambda \sim R^{-1}$  are suppressed, in the regularized FRR formalism this parameter simply becomes a scale beyond which the effective theory is no longer valid. That is, the parameter  $\Lambda$  no longer strictly characterizes a physical property of a model. We thus assert that  $\Lambda_H = \Lambda_\Lambda = \Lambda$  is an appropriate choice.

### 3.3 Variations of the phenomenological inputs

Certain phenomenologically determined constants were input into the fit of the binding of the  $H$ -dibaryon. In particular,  $f$ , the meson decay constant in the chiral limit, and the baryon-baryon-meson coupling constants,  $F$  and  $C$ , were set to values determined by chiral perturbation theory and SU(6) symmetry.

We test the robustness of the method by varying these parameters by  $\pm 10\%$ , and noting that the dependence of the binding of the  $H$  on these variations is small compared to the statistical error associated with our fit. We find the  $H$ -dibaryon unbound by 12 to 14 MeV, 9 to 17 MeV, and 11 to 15 MeV at the physical point, for 10% variations in  $f$ ,  $F$  and  $C$  respectively. There is little associated variation in the  $\chi^2$  values for these fits, all lying between 0.44 and 0.5 (compared to 0.48 for the primary fit). We also note that the values of the octet baryon masses from each of these fits are compatible, within quoted error, with those of the primary fit. Table 3 gives best-fit parameters for each of the fits considered.

### 3.4 Variation of the chiral coefficient $C_H$

Figure 1 and Table 4 give the results of performing fits to the octet and binding data, while holding  $C_H \sim 4.1$  fixed at the Mulders-Thomas value [19], calculated using SU(6) symmetry. As is clear from Figure 1, the fit is not as good as the primary results. In fact, there is a  $\chi^2$  of almost 2 per degree of freedom associated with the binding portion of the fit. At the physical point, the  $H$ -dibaryon is unbound by  $30 \pm 9$  MeV.

**Table 3** Best-fit parameters for the octet and  $H$ -dibaryon lattice data, setting  $f$ ,  $F$  and  $C$  to be 10% smaller or larger than their phenomenological values, given in Ref. [20]. All quantities are given in appropriate powers of GeV.

(a) Vary $f$ , the meson decay constant in the chiral limit.								
$\mathbf{f} \rightarrow \mathbf{0.9f}$	$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$	$\sigma$	$B_0$	$\sigma_B$	$C_H$
best fit value	0.92	0.85	-1.86	-1.28	-0.58	0.015	-2.40	7.03
error	0.06	0.04	0.13	0.11	0.06	0.005	0.22	0.11
$\mathbf{f} \rightarrow \mathbf{1.1f}$								
best fit value	1.12	0.87	-1.59	-1.13	-0.46	0.08	-1.25	6.29
error	0.14	0.04	0.11	0.09	0.05	0.02	0.14	0.02
(b) Vary the baryon-baryon-meson coupling constant $F$ .								
$\mathbf{F} \rightarrow \mathbf{0.9F}$	$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$	$\sigma$	$B_0$	$\sigma_B$	$C_H$
Best fit value	1.05	0.86	-1.60	-1.26	-0.48	0.027	-2.45	5.39
error	0.12	0.03	0.11	0.10	0.05	0.002	0.21	0.08
$\mathbf{F} \rightarrow \mathbf{1.1F}$								
best fit value	0.99	0.86	-1.81	-1.12	-0.55	0.011	-2.24	5.93
error	0.12	0.04	0.13	0.09	0.06	0.005	0.19	0.09
(c) Vary the baryon-baryon-meson coupling constant $C$ .								
$\mathbf{C} \rightarrow \mathbf{0.9C}$	$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$	$\sigma$	$B_0$	$\sigma_B$	$C_H$
best fit value	1.06	0.86	-1.71	-1.14	-0.51	0.013	-2.24	5.91
error	0.13	0.04	0.12	0.09	0.05	0.005	0.19	0.10
$\mathbf{C} \rightarrow \mathbf{1.1C}$								
best fit value	0.98	0.86	-1.71	-1.25	-0.51	0.025	-2.50	5.41
error	0.11	0.04	0.12	0.11	0.05	0.003	0.21	0.08

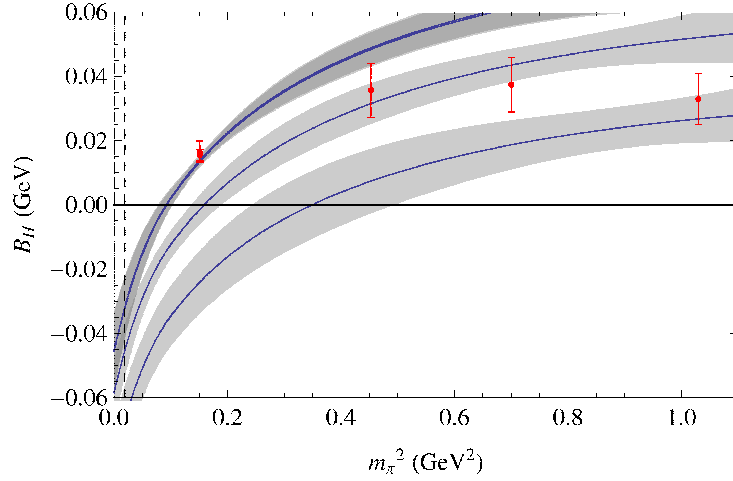
**Table 4** Values of the fit parameters for the octet and  $H$ -dibaryon data corresponding to the fit shown in Figure 1. Note that units are given in dimensionally appropriate powers of GeV.

	$\Lambda$	$M^{(0)}$	$\alpha$	$\beta$
best fit value	0.77	0.93	-1.46	-0.98
error	0.06	0.03	0.13	0.10
	$\sigma$	$B_0$	$\sigma_B$	
	-0.40	0.06	-1.20	
	0.06	0.03	0.08	

## 4 Conclusion

We have briefly reviewed the experimental and theoretical status of the strange vector form factors of the proton. Their importance is that since they involve only “disconnected diagrams” the capacity of non-perturbative QCD to calculate them is comparable to that of the successful calculation of the Lamb shift in QED. As we showed, QCD passes this test with flying colours.

With regard to the strange scalar form factor, there has been a dramatic downward shift in the past couple of years, with the contribution to the mass of the nucleon arising through the strange



**Fig. 1** Binding energy of the  $H$ -dibaryon versus pion mass squared, resulting from our chiral fit with  $C_H$  fixed at the Mulders-Thomas value [19], for several values of the kaon mass at which the simulations by HAL QCD and NPLQCD were carried out.

quark mass now around 3%, rather than 30%. As we explained, this is critical to the interpretation of searches for neutralino dark matter.

Finally, we pointed out the very exciting new developments concerning the  $H$ -dibaryon, which now appears to be only slightly unbound. Although the calculation of Shanahan *et al.* [20], which found the  $H$  unbound by  $13 \pm 14$  MeV assumed that it was a true multi-quark state, similar values have since been reported under quite different assumptions [21, 22]. This makes it urgent to pursue, initially at J-PARC, the sort of work reported in Ref. [23], which has already given a hint of the existence of such a state.

**Acknowledgements** This work was supported by the University of Adelaide and by the Australian Research Council through grants FL0992247 (AWT) and DP110101265 (RDY).

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